SENSITIVITY ANALYSIS OF A MODERN AUTOMATIC CONTROL SYSTEM FOR THE ACTIVATED SLUDGE PROCESS IN WASTEWATER TREATMENT

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EXTENDED ABSTRACT

This study is about the sensitivity analysis of Modern Automatic Control (MAC) system of Activated Sludge Process (ASP) which is widely used in wastewater treatment. This process deals with severe variations of the incoming wastewater flow rate \( q_F \) that could make it fail. Urban wastewater flow rate is difficult to be predicted in comparison with industrial wastewater flow rate which either has significantly less fluctuations or the fluctuations are predictable. Also two other factors that are making \( q_F \) to change (increase) significantly is rainfall and tourism (or more generally population changes). ASP removes the organic substances that are dissolved in the wastewater. It accomplishes this by using aerobic microorganisms that consume these substances and produce carbon dioxide and water in their place. The process is constituted by a bioreactor or aeration tank, a sedimentation tank or clarifier and a recirculation pump that feeds the microorganisms or sludge back to the bioreactor so their concentration to be effective for organic substances removal. The clarifier overflow is the treated wastewater which is safely discharged in the environment. A portion of the sludge is discharged for further treatment. ASP failure causes significant threat to the ecosystem. The aim of the MAC system is to make the process more resistant to the fluctuations of the influent wastewater. MAC is a type of control system which uses the feedback of state variables that characterize the process to control ASP. The proposed MAC method takes into account the multivariability of the ASP. In this study a sensitivity analysis has been done for MAC system. The aim is to see the control system’s sensitivity in parameter changes. Sensitivity Function (SF) is the ratio of the percent change of the system transfer function to the percent change of a parameter \( b \) of the transfer function. The parameter for which sensitivity of the system is measured is influent flow rate \( q_F \). After calculation of the SF it is shown that sensitivity is high for frequencies in a region around \( 60 \) rad/day (the resonant frequency) which is the same for oxygen, substrate and microorganisms subsystems. Additionally in the steady state (zero frequency), none of the three subsystems has steady state error.

Keywords: Activated sludge process, modern automatic control, sensitivity analysis, wastewater treatment.

1. INTRODUCTION

ASP is used in wastewater treatment plants and it is considered to be a very important process for the pollutant removal (Tchobanoglous, 2002). This process has to deal with the fluctuations in the influent wastewater flow rate which is denoted as \( q_F \). Wastewater treatment is very effective with this method (Olsson and Newell, 1999). ASP is shown in Figure 1 where the influent substrate concentration is \( S_{in} \). The major problem with \( q_F \) is that if it is increased rapidly it washes out all the microorganisms of the bioreactor and the process fails, allowing the wastewater to be discharged untreated and to degrade the environment and human health. After this washout the ASP is very difficult to start.
functioning because significant time is needed for the microorganisms to increase their concentration in the bioreactor (Tchobanoglous, 2002; Olsson and Newell, 1999).

Figure 1. The ASP.

2. THE AUTOMATIC CONTROL SYSTEM OF THE PROCESS

For many years the automatic control of ASP had been fulfilled with classical control through P or PI type controllers (Wahab, 2007). This is a simple and cheap solution for the process control. MAC control has many and important advantages over classic control. It is a more technologically advanced system. MAC uses the feedback of state variables that fully describe the ASP as a mean to control ASP (Ogata, 2009; Phillips and Harbor, 2000). MAC is a new method of control and further study has to be done in order to measure the control system’s robustness (Lee, 2003; Francisco, 2011; Ekman, 2005). Sensitivity analysis is such a field that must be studied.

The model that is used for describing ASP in this study is composed of three differential equations which are the mass balances of substrate, dissolved oxygen and microorganisms (Olsson and Newell, 1999; Xiongwei, 2010). The control variables are the $q_W$ (sludge discharge flow rate), $q_R$ (recycled sludge flow rate) and $q_A$ (air flow rate). The controlled variables or state variables are the $S_S$ (substrate concentration), $S_O$ (dissolved oxygen concentration) and $X_H$ (microorganisms concentration). The basic assumptions that were made are a) the clarifier is operated ideally, b) the state variables are measurable with precision and in real time and c) only organic carbon removal is considered (Olsson and Newell, 1999; Tchobanoglous, 2002).

A control system is required to respond in some controlled manner to inputs and this is called stability. Since an exact model of a physical system is never available the characteristics of a closed-loop control system should be insensitive to the parameters of the mathematical model which is used in the design of the control system (Ludovic, 2004). The parameters of the ASP model will possibly change with time and it is desired the control system’s characteristics to be insensitive to these changes (Alfaro, 2008). This is called sensitivity of the system.

3. THE SENSITIVITY OF THE SYSTEM

3.1. Sensitivity calculation

Sensitivity analysis shows how the control system reacts in the variation of model’s parameters. Usually for sensitivity measurement the following equation is used:

$$S = \frac{\Delta T(s)}{T(s)}$$

Where $S$ is the Sensitivity Function (SF), $T(s)$ is the system’s transfer function (control system with ASP) and $b$ a parameter of $T(s)$ that is changed in order to see the transfer
function change. This is called sensitivity of the system’s transfer function for the $b$ parameter of the transfer function. From the above equation it is seen that the ratio of the percent change in the system transfer function to the percent change in a parameter $b$ of the transfer function gives the sensitivity $S_b$. So sensitivity can be expressed as $S_b^T$. The sensitivity function is evaluated in the limit as $\Delta b$ approaches zero (Phillips and Harbor, 2000).

$$S_b^T = \lim_{\Delta b \to 0} \frac{\Delta T(s)}{\Delta b} \cdot \frac{b}{T(s)} \cdot \frac{\partial T(s)}{\partial b} \cdot \frac{b}{T(s)}$$

The SF is a function of the Laplace transform variable $s$, so the interpretation of sensitivity is very difficult. For this reason the variable $s$ is replaced with $j \cdot \omega$ and the sensitivity becomes a frequency response (Phillips and Harbor, 2000). Then meaning can be assigned to sensitivity for frequencies in the bandwidth of the system.

Sensitivity helps in spotting the weaknesses of the system. That is to find which parameter variation affects the system more and take precautions to make the system more insensitive. So it is a valuable tool for the control system designer to take extra care and modify the design in such a way that the system can face changes in those parameters in which it is sensitive (Alfaro et al, 2008; Francisco et al, 2011).

### 3.2. Frequency response

Frequency response of a system is its steady state response in a sinusoidal input of the form $r(t) = A \cdot \cos(\omega \cdot t)$. The system has a transfer function of $T(s)$ and to get its frequency response $s$ is substituted with $\omega \cdot j$. For a given value of $\omega$ the transfer function $T(\omega \cdot j)$ is a complex number and it is expressed as:

$$T(\omega \cdot j) = |T(\omega \cdot j)| \cdot e^{j\phi(\omega)}$$

In the above equation $|T(\omega \cdot j)|$ is the magnitude of the transfer function’s frequency response and $\phi$ is the angle of the complex number (phase shift) of the output sinusoid (Phillips and Harbor, 2000).

The gain of a stable linear time – invariant system to a sinusoidal input of frequency $\omega$ in both magnitude and phase is given by the transfer function evaluated at $s = \omega \cdot j$. Using the system of Figure 2 the following equation is applied: $C(s) = T(s) \cdot E(s)$ (Phillips and Harbor, 2000).

![Figure 2. Stable linear and time – invariant system.](image)

Then $s$ is substituted with $\omega \cdot j$ and for a given $\omega$, $T(\omega \cdot j)$ is a complex number and it is written as: $T(\omega \cdot j) = |T(\omega \cdot j)| \cdot e^{j\theta(\omega)}$. $\theta$ is the phase shift which was mentioned above.
as \( \phi \). If the input signal \( e(t) \) is sinusoidal: \( e(t) = A \cdot \sin(\omega \cdot t) \). Then the output signal in steady state \( (\omega = 0 \ \text{rad} / \text{day}) \) is \( c_s(t) \) and its value is:
\[
c_s(t) = A \cdot T(\omega \cdot j) \cdot \sin[\omega \cdot t + \theta(\omega)]
\]

3.3. System’s bandwidth

The bandwidth \( \omega_b \) of a system is the frequency that reduces the system’s magnitude to 0.707 or \( \sqrt{\frac{1}{2}} \) of the magnitude in zero frequency (steady state) \( \sqrt{\frac{1}{2}} \cdot T(j \cdot 0) \) or 0.707 \( T(j \cdot 0) \) (Phillips and Harbor, 2000). These frequencies are 494 \( \text{rad} / \text{day} \) for \( S_O \), 100.5 \( \text{rad} / \text{day} \) for \( S_S \) and 105.2 \( \text{rad} / \text{day} \) for \( X_H \).

For frequencies that are outside the system’s bandwidth, sensitivity is not important because such frequencies will never be applied to the system. In general it is true that the system is very sensitive in parameter changes of the process model but outside the system’s bandwidth so this is not of much concern. Additionally the system is sensitive in the gain of the sensor that sends the feedback signal inside the bandwidth so this is important and special care must be given in the quality of the sensor (Phillips and Harbor, 2000).

To find the bandwidths of the three subsystems the transfer functions must be known. The transfer functions have been calculated from the mathematical model of ASP (Olsson and Newell, 1999) which was linearized (Phillips and Harbor, 2000) and its block diagram was modified in order to be controlled from a MAC system (Phillips and Harbor, 2000). When the three transfer functions are acquired \( s \) is substituted with \( \omega \cdot j \) from which \( \omega = 0 \ \text{rad} / \text{day} \) (Phillips and Harbor, 2000).

These transfer functions are derived from the system for \( q_F = 50 \ (\text{ML} / \text{day}) \) and give the following values for \( \omega = 0 \ \text{rad} / \text{day} \) (the system’s type is 0).

\[
S_{\text{Odenominator}}(0) = 6206096947914546.829
\]

\[
S_{\text{Onumerator}}(0) = 3094559626239382.78
\]

\[
|T_{S_O}(0)| = \left| \frac{S_{\text{Onumerator}}(0)}{S_{\text{Odenominator}}(0)} \right| = \frac{3094559626239382.78}{6206096947914546.829} = 0.499 \ \text{mg} / \text{l}
\]

\( S_O \) magnitude reduced value: \( \frac{0.499}{\sqrt{2}} = 0.353 \ \text{mg} / \text{l} \)

\[
S_{\text{Snumerator}}(0) = -8734867013927458.024
\]

\[
|T_{S_S}(0)| = \left| \frac{S_{\text{Snumerator}}(0)}{S_{\text{Odenominator}}(0)} \right| = \frac{8734867013927458.024}{6206096947914546.829} = 0.147 \ \text{mg} / \text{l}
\]
$$S_s \text{ magnitude reduced value: } \frac{0.147}{\sqrt{2}} = 0.104 \text{ mg} / l$$

$$X_{\text{numerator}}(0) = 4760504583767732.58$$

$$\left| T_{x_n}(0) \right| = \left| \frac{X_{\text{numerator}}(0)}{S_{\text{denominator}}(0)} \right| = \frac{4760504583767732.58}{6206096947914546.829} = 0.767 \text{ mg} / l$$

$$X_H \text{ magnitude reduced value: } \frac{0.767}{\sqrt{2}} = 0.542 \text{ mg} / l$$

4. RESULTS

4.1. Bandwidths of the three subsystems

The three subsystems bandwidths are calculated from their magnitudes in zero frequency when they are reduced to 0.707 or $\frac{1}{\sqrt{2}}$ of their original value as is shown in the above equations and then the frequency is adjusted in such a way that the magnitude of their transfer functions is equal to their reduced values. These frequencies are the subsystems bandwidths $\omega_B$ (see Figures 3, 4 and 5). So the frequency $\omega_B$ denotes the frequency at which the gain is equal to $\frac{1}{\sqrt{2}}$ times the gain at very low frequencies. This frequency $\omega_B$ is called bandwidth of the system (Phillips and Harbor, 2000).

**Figure 3.** Bandwidth frequency region $\omega_B$ estimation for $S_O$.

**Figure 4.** Bandwidth frequency region $\omega_B$ estimation for $S_S$.

**Figure 5.** Bandwidth frequency region $\omega_B$ estimation for $X_H$. 
Magnitudes in the resonant frequency $\omega = 60 \text{ rad/day}$ are $31.374 \text{ mg/l}$ for $S_O$, $1.113 \text{ mg/l}$ for $S_S$ and $6.36 \text{ mg/l}$ for $X_H$. Additionally from Figures 3, 4 and 5 it is shown that the $S_O$ subsystem has 5 times larger bandwidth in comparison with the other subsystems, because of the faster dynamics that this subsystem has.

### 4.2. Sensitivity analysis

A sensitivity analysis of the system can be done with the help of the diagrams below (Figures 6, 7 and 8), (Rauh, 2009). These diagrams show the sensitivity of the system’s transfer functions in the sinusoidal fluctuations of the parameter $q_F$ (of the transfer function) which has a nominal value of $50 \text{ ML/day}$ (Phillips and Harbor, 2000). In the rest of this study wherever sensitivity is mentioned, it is meant sensitivity of the system’s transfer function in the fluctuations of the parameter $q_F$ (of the transfer function).

For $S_O$ the sensitivity diagram shows a strong effect of $q_F$ by increasing the subsystem’s sensitivity especially in the region of $\omega = 60 \text{ rad/day}$ where there is a resonance (see Figure 6). Below and beyond resonant frequency and inside system’s bandwidth, the sensitivity value is small. So $S_O$ is a sensitive state variable in the sinusoidal fluctuations of $q_F$ by exceeding 23 (sensitivity value) in its resonant frequency. This value means that in a 10% increase in $q_F$ sinusoidal fluctuation magnitude the $S_O$ magnitude will be 25.3 times larger so it will be increased by 2430%: $S_O = 23 \cdot 1.1 = 25.3$ or $24.3 \cdot 100 = 2430\%$

For $S_S$ and $X_H$ the sensitivity diagrams show a larger effect of $q_F$ in their sensitivities by exceeding 70 for the same resonant frequency of $\omega = 60 \text{ rad/day}$ (Figures 7 and 8). This value means that in a 10% increase in $q_F$, the $S_S$ and $X_H$ magnitudes will be 70 times larger so they will be increased approximately by 7600%.

In steady state ($\omega = 0 \text{ rad/day}$) all three subsystems have zero sensitivity values and this means that they don’t have steady state error (Phillips and Harbor, 2000). For small values of frequency ($\omega$) sensitivity of the three subsystems is small. As frequency increases there is a resonance in $\omega = 60 \text{ rad/day}$ for all three subsystems and in the end of their frequency bandwidths (as their frequencies approach $\omega_B$) all three subsystems’ sensitivity to $q_F$ becomes smaller (see Table I). In Table I frequency regions are shown that have sensitivity values lower than 10. Sensitivity value of 10 means that for an increase of 10% in $q_F$ magnitude the increase in the magnitudes of $S_O$, $S_S$ and $X_H$ is ten times larger.
Figure 6. Sensitivity of the subsystem $S_O$ in $q_F$.

Figure 7. Sensitivity of the subsystem $S_S$ in $q_F$.

Figure 8. Sensitivity of the subsystem $X_H$ in $q_F$.

Table I. Low sensitivity frequency regions

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Sensitivity lower than 10 in low frequency region</th>
<th>Sensitivity lower than 10 in high frequency region</th>
</tr>
</thead>
<tbody>
<tr>
<td>For $S_O$</td>
<td>$0 \rightarrow 40 \text{ rad/day}$</td>
<td>$120 \rightarrow 495 \text{ rad/day}$</td>
</tr>
<tr>
<td>For $S_S$</td>
<td>$0 \rightarrow 40 \text{ rad/day}$</td>
<td>Not in the bandwidth region</td>
</tr>
<tr>
<td>For $X_H$</td>
<td>$0 \rightarrow 40 \text{ rad/day}$</td>
<td>Not in the bandwidth region</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

In this study it is shown that for an increase of 10 % in $q_F$ magnitude the sensitivity of the system is large in the resonant frequency region. However in the rest of the bandwidth MAC is a robust system which can withstand changes of the model parameter. $q_F$ is a strong parameter of the model and the relatively low sensitivity values in the low and high frequency regions show the high robustness of the system. Moreover none of the three subsystems has steady state error and this shows the high robustness of the MAC system.

It is shown that among the state variables those that were most influenced, (those which had more sensitivity) are the $S_S$ and $X_H$. For $S_S$ it is easy to understand that with the same $q_W$ there is an increase in substrate concentration because of the increase in the $q_F$ which has an elevated $S_{SF}$ concentration. The opposite thing happens with $X_H$. 
In this study a sensitivity analysis is carried out for the system of ASP that is controlled by the MAC system. The calculations show that this system is relatively robust in $q_F$ fluctuations. An opportunity is given to compare the sensitivities between state variables and take precaution measures. In this study the sensitivity analysis was done only for parameter $q_F$. Another parameter that could be studied is $S_{SF}$ (influent substrate concentration).

REFERENCES

7. Olsson G. and Newell B. (1999), Wastewater Treatment Systems Modelling, Diagnosis and Control, IWA Publishing.