FUZZY MULTI-CRITERIA DECISION MAKING METHOD FOR DAM SELECTION.

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EXTENDED ABSTRACT

This research formulates the process of a dam selection (in the district of Chalkidiki in Northern Greece) as a multi-criteria analysis problem. The TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution) is chosen among the several methods which are used for multi-criteria decision making. The principle of TOPSIS method is that the chosen alternative should be as close to the ideal solution as possible and at the same time as far as possible from the negative-ideal solution.

For the application, certain criteria have been selected for each alternative and as the optimal solution, the alternative showing the greatest performance was considered. The selected criteria are five: a) $x_1$ = hydrology, b) $x_2$ = geology, c) $x_3$ = environment, d) $x_4$ = hydropower and e) $x_5$ = total cost. The ratings of each alternative and the weight of each criterion represents uncertainties and were described by linguistic terms which can be expressed in triangular fuzzy numbers. As a second approach, there has been a proposition of evaluating the criteria’s weights using an objective method, meaning Shannon's entropy.

KEYWORDS: Optimal dam selection, TOPSIS method, fuzzy weight’s assessment, multi-criteria decision making, linguistic variables, Shannon entropy.

1. INTRODUCTION

The Olynthios river passes through the central part of the Chalkidiki peninsula in Northern Greece (Fig. 1). The dam will be constructed to supply potable water for the urban areas as well as to fulfill agricultural purposes. Three sites (‘Tsaousi Milos’ station, ‘Psalida’ station, ‘Louziki’ station) and three types (rock fill dam, rock fill dam with lining of clay to the right bank and roller-compacted concrete) for the dam were considered.
Our attempt to choose the optimal dam site and type selection is actually a Multiple Criteria Decision Making problem. Several techniques are available such as the Compromise Programming (Zeleny, 1974), the Analytic Hierarchy Process (Saaty, 1980), the Cooperative Game Theory (Nash, 1953; Szidarovszky et al., 1984), the Composite Programming (Bardossy et al., 1985) etc. Among those techniques we selected the TOPSIS method into fuzzy environment, using a linguistic scaling (Chen, 2000) and Shannon’s entropy, which are briefly presented as it follows.

2. TOPSIS (TECHNIQUE FOR ORDER PREFERENCE BY SIMILARITY TO IDEAL SOLUTION) METHOD

The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was first developed at 1981 by Hwang and Yoon (Hwang et al., 1981). Its basic concept is that the chosen alternative should have the shortest distance from the ideal solution and the farthest from the negative-ideal solution. The procedure of fuzzy TOPSIS is similar to the classic one and can be expressed in a series of steps:

a) Construct the normalized decision matrix, after obtaining the initial decision matrix.
   – In the fuzzy environment, in order to avoid the complicated normalization formula used in classical TOPSIS, the linear scale transformation is used to transform the various criteria scales into a comparable scale.

\[
\tilde{r}_{ij} = \left( \frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^-}, \frac{c_{ij}}{c_j^*} \right), \quad c_j^* = \max_i c_{ij} \tag{2.1}
\]

where \(\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})\) are the elements of the initial decision matrix.

b) Construct the weighted normalized decision matrix.

\[
\tilde{v}_{ij} = \tilde{w}_j \cdot \tilde{r}_{ij}, \quad j=1,2,\ldots,m, \quad i=1,2,\ldots,n \tag{2.2}
\]

c) Determine the fuzzy ideal and fuzzy negative-ideal solutions.
\[ A^+ = \{ \tilde{v}^+_1, \tilde{v}^+_2, ..., \tilde{v}^+_m \} \]  
\[ A^- = \{ \tilde{v}^-_1, \tilde{v}^-_2, ..., \tilde{v}^-_m \} \]

where \( \tilde{v}^+_j = (1,1,1) \) and \( \tilde{v}^-_j = (0,0,0) \), \( j=1,2,...,m \).

d) Calculate the separation measure:
- Ideal separation
  \[ S_i^+ = \sum_{j=1}^{m} s(\tilde{v}_{ij}, \tilde{v}^+_j) \quad i=1,2,...n \]  
- Negative-ideal separation
  \[ S_i^- = \sum_{j=1}^{m} s(\tilde{v}_{ij}, \tilde{v}^-_j) \quad i=1,2,...n \]  

where \( s(\tilde{v}_{ij}, \tilde{v}^+_j) \) and \( s(\tilde{v}_{ij}, \tilde{v}^-_j) \) are distance measurements.

e) Calculate the relative closeness to the Ideal Solution.
\[ c_i^* = \frac{S_i^-}{(S_i^+ + S_i^-)} , \quad 0 < c_i^* < 1, \quad i=1,2,...,n \]  
\[ c_i^* = 1 \quad \text{if} \quad A_i = A^* \]  
\[ c_i^* = 0 \quad \text{if} \quad A_i = A^- \]  
f) Rank the preference order.
- A set of alternatives can now be preference ranked according to the descending order of \( c_i^* \).

![Figure 2: Basic concept of TOPSIS method (A*: Ideal point, A': Negative-Ideal Point).](image)

The method assumes that:

- a. Each criterion in the decision matrix takes either monotonically increasing or monotonically decreasing utility (Saaty, 1978).
- b. A decision matrix of \( n \) alternatives and \( m \) criteria and a set of weights for the criteria are available.
- c. Any outcome which is expressed in a non-numerical way should be quantified through the appropriate scaling technique (Chen, 2000).

3. LINGUISTIC VARIABLES EXPRESSED IN TFN (TRIANGULAR FUZZY NUMBERS)
The extension of the TOPSIS method in the fuzzy environment can be achieved by expressing the weights of the criteria and the ratings as linguistic variables. A linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions (Zadeh, 1975). According to many authors, the linguistic variables can be expressed in positive triangular fuzzy numbers as shown in Table 1 (Chen, 2000).

### TABLE 1. Linguistic variables.

<table>
<thead>
<tr>
<th>Linguistic variables for the importance weight of each criterion</th>
<th>Linguistic variables for the ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (VL)</td>
<td>(0,1,0,0)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.3,0.1,0.1)</td>
</tr>
<tr>
<td>Medium low (ML)</td>
<td>(0.5,0.3,0.3)</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>(0.7,0.5,0.5)</td>
</tr>
<tr>
<td>Medium high (MH)</td>
<td>(0.9,0.7,0.7)</td>
</tr>
<tr>
<td>High (H)</td>
<td>(1,0.9,0.9)</td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>(1,1,1)</td>
</tr>
</tbody>
</table>

### 4. TRIANGULAR FUZZY NUMBERS (TFN)

We define a fuzzy number $\tilde{M}$ on $R^+$ to be a triangular fuzzy number if its membership function $\mu_{\tilde{M}}(x): R \to [0, 1]$ is equal to:

$$
\mu_{\tilde{M}}(x) = \begin{cases} 
\frac{x - \ell}{m - \ell} & x \in [\ell, m] \\
\frac{x - u}{m - u} & x \in [m, u] \\
0 & \text{otherwise}
\end{cases},
$$

where $\ell \leq m \leq u$, and $\tilde{M} = (\ell, m, u)$.

[Figure 3: A triangular fuzzy number $\tilde{M}$.]

### 5. SYNTHETIC EVALUATION METHOD

In order to avoid the uncertainty associated with the mapping of one’s judgment to a number, we propose an objective evaluation method for the weights of the criteria. Entropy is a good measure to be used for evaluating the relative worthiness of different tests or assessment procedures (Zeleny, 1982).

Consider $X=[x_{ij}]_{n \times m}$ to be the matrix which presents the measured scores of $j$ ($j=1,2,\ldots,m$) criteria for $i$ ($i=1,2,\ldots,n$) alternatives. Secondly, we transform the $x_{ij}$ into degrees of closeness to the point $x^* = [\max_{i=1}^n (x_{i1}), \max_{i=1}^n (x_{i2}), \ldots, \max_{i=1}^n (x_{im})]$. Using $d_{ij} = x_{ij} / x^*$, we obtain the matrix $D=[d_{ij}]_{n \times m}$. The vector $d_j = (d_{1j}, d_{2j}, \ldots, d_{nj})$ characterizes the set D in terms of the $j$th criterion. Then we calculate the entropy measure $e(d_j)$ of each criterion as it follows.
\[ e(d_j) = -K \sum_{i=1}^{n} d_{ij} \ln \frac{d_{ij}}{D_j} \] where \( K = \frac{1}{e_{\text{max}}} \) and \( e_{\text{max}} = \ln n \).  

(5.1)

The total entropy of \( D \) is defined as:

\[ E = \sum_{j=1}^{m} e(d_j) \]

(5.2)

Because weights \( w_j \) are reversely related to \( e(d_j) \), we shall use \( 1 - e(d_j) \) rather than \( e(d_j) \) and normalize to assure that \( 0 \leq w_j \leq 1 \) and \( \sum_{j=1}^{m} w_j = 1 \): \[
w_j = \frac{1}{m - E} [1 - e(d_j)] \]

(5.3)

This method presupposes a non-fuzzy decision matrix. In a fuzzy environment, we could use a process called defuzzification, such as the centroid technique (Mamdani, 1976).

6. ILLUSTRATIVE APPLICATION

In this paper we use the TOPSIS method into fuzzy environment and two methods for the assessment of the weights: a) by using linguistic variables for the importance of each criterion and b) by using Shannon’s entropy. First we use each method for the assessment of the weights as well as the overall performance of the alternatives in each criterion. In second step these performances and weights are considered and used in TOPSIS process. Then TOPSIS is applied for the evaluation problem and the result shows the preference order.

There are five criteria: a) \( x_1 = \) hydrology, b) \( x_2 = \) geology, c) \( x_3 = \) environment, d) \( x_4 = \) hydropower and e) \( x_5 = \) total cost and five alternatives (stations): a) \( X_1 = \) ‘Tsaousi Milos’ station- rock fill dam, b) \( X_2 = \) ‘Psalida’ station- rock fill dam with lining of clay to the right side, c) \( X_3 = \) ‘Psalida’ station- rock fill dam, d) \( X_4 = \) ‘Louziki’ station- rock fill dam and e) \( X_5 = \) ‘Louziki’ station- RCC (Roller-compacted concrete) dam.

The decision matrices with linguistic and quantitative expressions for the variables where taken indirectly from the article “Multi-criteria evaluation of alternative dam sites and head works of Olynthios river in Chalkidiki” (Parisopoulos et al., 2011):

<table>
<thead>
<tr>
<th>Criteria</th>
<th>X_1</th>
<th>X_2</th>
<th>X_3</th>
<th>X_4</th>
<th>X_5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternatives</td>
<td>( G )</td>
<td>( G )</td>
<td>( P )</td>
<td>( VP )</td>
<td>( VG )</td>
</tr>
<tr>
<td>( X_1 )</td>
<td>9,10,10</td>
<td>7,9,10</td>
<td>0,1,3</td>
<td>0,0,1</td>
<td>9,10,10</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>9,10,10</td>
<td>5,7,9</td>
<td>3,5,7</td>
<td>3,5,7</td>
<td>5,7,9</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>9,10,10</td>
<td>9,10,10</td>
<td>3,5,7</td>
<td>3,5,7</td>
<td>1,3,5</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>9,10,10</td>
<td>9,10,10</td>
<td>7,9,10</td>
<td>9,10,10</td>
<td>7,9,10</td>
</tr>
<tr>
<td>( X_5 )</td>
<td>9,10,10</td>
<td>7,9,10</td>
<td>7,9,10</td>
<td>9,10,10</td>
<td>5,7,9</td>
</tr>
</tbody>
</table>

The normalized decision matrix, using the equation 2.1:
Matrix 2

\[
\begin{bmatrix}
X_1 & 0.7,0,9,1.0 & 0.7,0,9,1.0 & 0.0,0,1,0.3 & 0.0,0,0.0,1 & 0.9,1,0,1.0 \\
X_2 & 0.9,1,0,1.0 & 0.5,0,7,0.9 & 0.3,0,5,0.7 & 0.3,0,5,0.7 & 0.5,0,7,0.9 \\
X_3 & 0.9,1,0,1.0 & 0.9,1,0,1.0 & 0.3,0,5,0.7 & 0.3,0,5,0.7 & 0.1,0,3,0.5 \\
X_4 & 0.7,0,9,1.0 & 0.7,0,9,1.0 & 0.7,0,9,1.0 & 0.9,1,0,1.0 & 0.7,0,9,1.0 \\
X_5 & 0.7,0,9,1.0 & 0.7,0,9,1.0 & 0.7,0,9,1.0 & 0.9,1,0,1.0 & 0.5,0,7,0.9
\end{bmatrix}
\]

6.1 Linguistic variables for the weights
The weights' matrices with linguistic and quantitative expressions for the criteria:

Matrix 3.1

<table>
<thead>
<tr>
<th>criteria</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(x_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MH</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>M</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>VH</td>
<td>0.9</td>
<td>1.1</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>H</td>
<td>0.7</td>
<td>0.9</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>VH</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

Matrix 3.2

<table>
<thead>
<tr>
<th>criteria</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(x_4)</th>
<th>(x_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[MH M VH H]</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>[0.63,0.9]</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>[0.70,0.9]</td>
<td>0.9</td>
<td>1.1</td>
<td>0.7</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>[0.7,0.9]</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.7,0.9]</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The weighted normalized decision matrix, using the equation 2.5:

Matrix 4

<table>
<thead>
<tr>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(x_3)</th>
<th>(x_4)</th>
<th>(x_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.63</td>
<td>0.90</td>
<td>0.21</td>
<td>0.45</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>0.81</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to the TOPSIS method, after the normalization of the decision matrix (Matrix 2) and the construction of the weighted normalized decision matrix (Matrix 4), the ideal separation, the negative- ideal separation and the relative closeness can be determined. By using the equations 2.5, 2.6 and 2.7 we have for \(S^+\), \(S^-\) and \(c_i^*\) respectively.

**TABLE 2. Ideal separation, negative- ideal separation and relative closeness.**

<table>
<thead>
<tr>
<th></th>
<th>(S_i^+)</th>
<th>(S_i^-)</th>
<th>(c_i^*)</th>
<th>normalized (c_i^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_1)</td>
<td>2.971</td>
<td>2.342</td>
<td>0.441</td>
<td>0.157</td>
</tr>
<tr>
<td>(X_2)</td>
<td>2.508</td>
<td>2.856</td>
<td>0.532</td>
<td>0.190</td>
</tr>
<tr>
<td>(X_3)</td>
<td>2.750</td>
<td>2.585</td>
<td>0.485</td>
<td>0.172</td>
</tr>
<tr>
<td>(X_4)</td>
<td>1.682</td>
<td>3.734</td>
<td>0.690</td>
<td>0.245</td>
</tr>
<tr>
<td>(X_5)</td>
<td>1.827</td>
<td>3.584</td>
<td>0.662</td>
<td>0.236</td>
</tr>
</tbody>
</table>

Finally, according to the relative closeness to the ideal solution \(c_i^*\), we rank the alternatives:

\(X_4 \succ X_5 \succ X_2 \succ X_3 \succ X_1\)

(The symbol '\(\succ\)' stands for 'better than'.)
6.2 Entropy method for the weights
After using the centroid defuzzification method to the decision matrix (Matrix 2), we can use the evaluation method of entropy. The entropy measure \( e(d_j) \) of each criterion is presented at the 5th matrix.

Matrix 5
(entropy)
\[
\begin{pmatrix}
x_1 & x_2 & x_3 & x_4 & x_5 \\
0.999 & 0.997 & 0.917 & 0.858 & 0.963 \\
\end{pmatrix}
\]

Matrix 6
(weights)
\[
\begin{pmatrix}
x_1 & x_2 & x_3 & x_4 & x_5 \\
0.003 & 0.012 & 0.314 & 0.533 & 0.138 \\
\end{pmatrix}
\]

After obtaining the values for the weights of the criteria (Matrix 6), we follow the procedure as it was described above.

Matrix 7
\[
\begin{pmatrix}
x_1 & x_2 & x_3 & x_4 & x_5 \\
0.002,0.003,0.003 & 0.009,0.011,0.012 & 0.000,0.031,0.094 & 0.000,0.000,0.053 & 0.125,0.138,0.138 \\
0.003,0.003,0.003 & 0.006,0.009,0.011 & 0.094,0.157,0.219 & 0.160,0.266,0.373 & 0.069,0.097,0.125 \\
0.003,0.003,0.003 & 0.011,0.012,0.012 & 0.094,0.157,0.219 & 0.160,0.266,0.373 & 0.014,0.042,0.069 \\
0.002,0.003,0.003 & 0.009,0.011,0.012 & 0.219,0.282,0.314 & 0.479,0.533,0.533 & 0.097,0.125,0.138 \\
0.002,0.003,0.003 & 0.009,0.011,0.012 & 0.219,0.282,0.314 & 0.479,0.533,0.533 & 0.069,0.097,0.125 \\
\end{pmatrix}
\]

TABLE 3. Ideal separation, negative-ideal separation and relative closeness.

<table>
<thead>
<tr>
<th></th>
<th>( S^+_i )</th>
<th>( S^-_i )</th>
<th>( C^*_i )</th>
<th>normalized ( C^*_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>4.794</td>
<td>0.236</td>
<td>0.047</td>
<td>0.075</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>4.475</td>
<td>0.557</td>
<td>0.111</td>
<td>0.177</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>4.527</td>
<td>0.507</td>
<td>0.101</td>
<td>0.162</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>4.082</td>
<td>0.925</td>
<td>0.185</td>
<td>0.296</td>
</tr>
<tr>
<td>( X_5 )</td>
<td>4.105</td>
<td>0.903</td>
<td>0.180</td>
<td>0.289</td>
</tr>
</tbody>
</table>

Finally, we have the ranking for the alternatives:
\( X_4 \succeq X_5 \succeq X_2 \succeq X_3 \succeq X_1 \)

7. CONCLUSIONS
The optimal solution using both the proposing methods, meaning fuzzy TOPSIS method with linguistic variables or objective evaluation (Shannon entropy) to assess the importance of the criteria is \( X_4 = \text{‘Louziki’ station- rock fill dam} \). In detail, we realize that the ranking according to both the procedures is the same, that is:
\( X_4 = \text{‘Louziki’ station- rock fill dam} \)
\( X_5 = \text{‘Louziki’ station- RCC (Roller-compacted concrete) dam} \)
\( X_2 = \text{‘Psalida’ station- rock fill dam with lining of clay to the right side} \)
\( X_3 = \text{‘Psalida’ station- rock fill dam} \)
\( X_1 = \text{‘Tsaousi Milos’ station- rock fill dam} \)
Even though the ranking doesn’t differ, it wouldn’t be right to assume that both approaches always lead us to the same results. In general we propose that these multi-criteria problems of great importance should be examined and solved, using multi-criteria methods established by the world literature and specifically multi-criteria methods in a fuzzy environment. Furthermore, we suggest that the chosen method should be followed also by different methods for the assessment of the weights which have either subjective or objective character.

Finally, we present you a graphic in which we can compare the results of the normalized relative closeness for each method, expressed as a percentage of the maximum value. We can see that there are significant differences as we are going to the least preferable option.

![Figure 4](image-url)  
**Figure 4**: A comparison of the relative closeness for each method.

**REFERENCES**